

DIRECT AND BENDING STRESSES

Combined bending and direct stresses

Prestress strength of material - Dr. R.K. Bansal

Consider the case of a column subjected by a compressive load p acting along the axis of the column.

This load will cause a direct compressive stress whose intensity will be uniform across the cross-section of the column.

σ_0 = Intensity of the stress

A = Area of cross-section

P = Load acting on the column.

then stress

$$\sigma_0 = \frac{\text{load}}{\text{Area}} = \frac{P}{A}$$

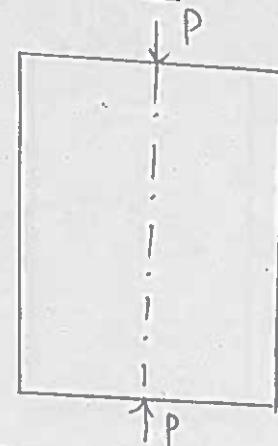
Consider the case of a column subjected by a compressive load p whose line of action is at a distance of e from the axis of the column as. Here e is known as eccentricity of the load. The eccentric load will cause direct and bending stress.

i) we have applied, along the axis of the column, two equal and opposite forces p .

thus three forces are acting now, on the column. one of the forces is shown in

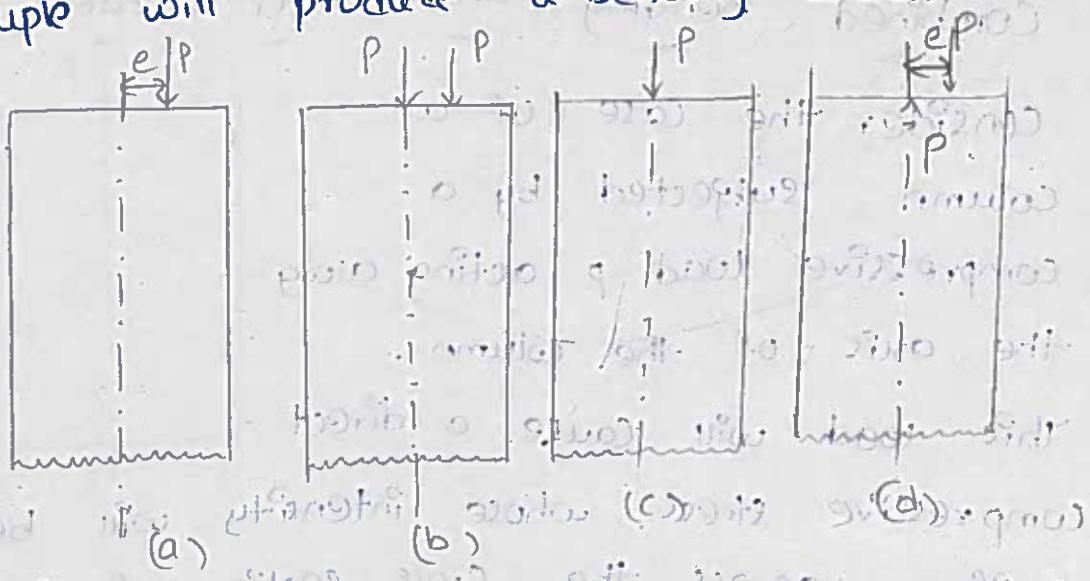
(c) and (d)

2) The force shown in fig (c) is acting along the axis of the column and hence this



force will produce a direct stress.

- 3) the forces shown in fig(d) will form a couple, whose moment will be Pxe . This couple will produce a bending stress.



Resultant Stress when a column of Rectangular section is subjected to an eccentric load

A column of rectangular section subjected to an eccentric load. Let the load is eccentric with respect to the axis 4-4. It is mentioned that an eccentric load cause direct stress as well as bending stress. Let us calculate these stresses.

P = Eccentric load on column

e = eccentricity of the load

σ_0 = Direct stress

σ_b = Bending stress

b = width of column

d = depth of column

Area of column section, $A = b \times d$

Now moment due to eccentric load P is given by,

$M = \text{load} \times \text{eccentricity}$

$M = Pxe$

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The direct stress (σ_0) is given by,

$$\sigma_0 = \frac{P}{A} \quad \text{--- (1)}$$

The bending stress (σ_b) due to moment at any point of the column section at a distance y from the neutral axis $Y-Y$ is given by

$$\frac{M}{I} = \frac{\sigma_b}{\pm y}$$

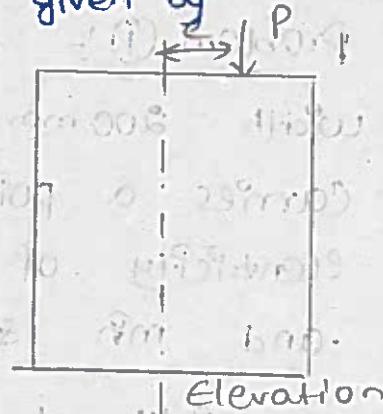
$$\sigma_b = \pm \frac{M}{I} \times y \quad \text{--- (2)}$$

$$I = \frac{d b^3}{12}$$

Sub in eqn (2), we get

$$\sigma_b = \pm \frac{M}{\frac{d \cdot b^3}{12}} \times y = \pm \frac{12M}{d \cdot b^3} \times y$$

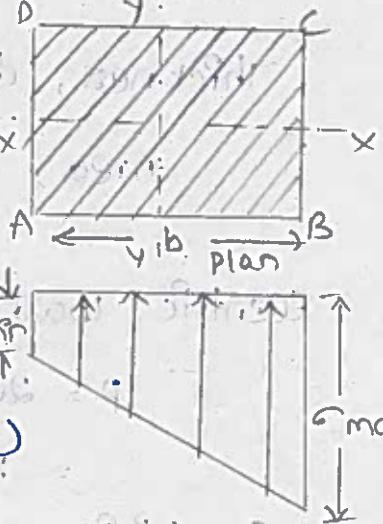
$$y = \frac{b}{2} \text{ in the above eqn.}$$



$$\therefore \sigma_b = \pm \frac{12M}{d \cdot b^3} \times \frac{b}{2} = \pm \frac{6M}{d \cdot b^2}$$

$$= \pm \frac{6Pxe}{d \cdot b^2} \quad (\because M = Pxe)$$

$$= \pm \frac{6Pxe}{d \cdot b \cdot b} = \pm \frac{6Pxe}{A \times b}$$



The resultant stress at any point will be the algebraic sum of direct stress and bending stress.

Let σ_{max} = max stress (i.e. stress along BC)

σ_{min} = min stress (i.e., stress along AD)

σ_{max} = Direct stress + Bending stress

$$= \sigma_0 + \sigma_b$$

$$= \frac{P}{A} + \frac{6Pxe}{A \cdot b}$$

$$= \frac{P}{A} \left[1 + \frac{6xe}{b} \right]$$

σ_{min} = Direct stress - Bending stress

$$= \sigma_0 - \sigma_b \quad \text{unit-3, pg.no-3/15}$$

$$= \frac{P}{A} - \frac{6P \cdot e}{A \cdot b} = \frac{P}{A} \left(1 - \frac{6e}{b} \right)$$

These stresses are shown in fig. The resultant stress along the width of the column will vary by a straight line law.

Problem ① :- A rectangular column of width 200 mm and of thickness 150 mm carries a point load of 240 kN at an eccentricity of 10mm. Determine the max and min stress on the section.

width $b = 200 \text{ mm}$

thickness, $d = 150 \text{ mm}$

$$\text{Area, } A = b \times d$$

$$= 200 \times 150 = 30000 \text{ mm}^2$$

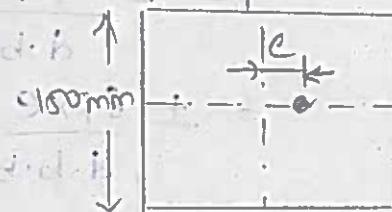
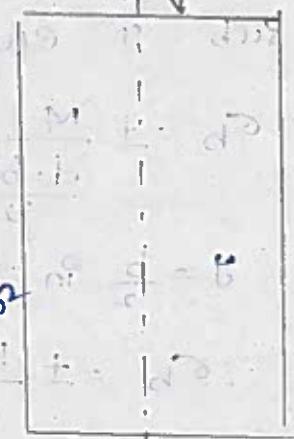
Eccentric load,

$$P = 240 \text{ kN}$$

$$= 240000 \text{ N}$$

Eccentricity,

$$e = 10 \text{ mm}$$



σ_{\max} = Max. stress

σ_{\min} = Min. stress

i) Using eqn, we get

$$\sigma_{\max} = \frac{P}{A} \left[1 + \frac{6e}{b} \right]$$

$$= \frac{240000}{30000} \left[1 + \frac{6 \times 10}{200} \right]$$

$$= 8 (1 + 0.3) = 10.4 \text{ N/mm}^2$$

ii) Using eqn, we get

$$\sigma_{\min} = \frac{P}{A} \left[1 - \frac{6e}{b} \right]$$

$$= \frac{240000}{30000} \left[1 - \frac{6 \times 10}{200} \right]$$

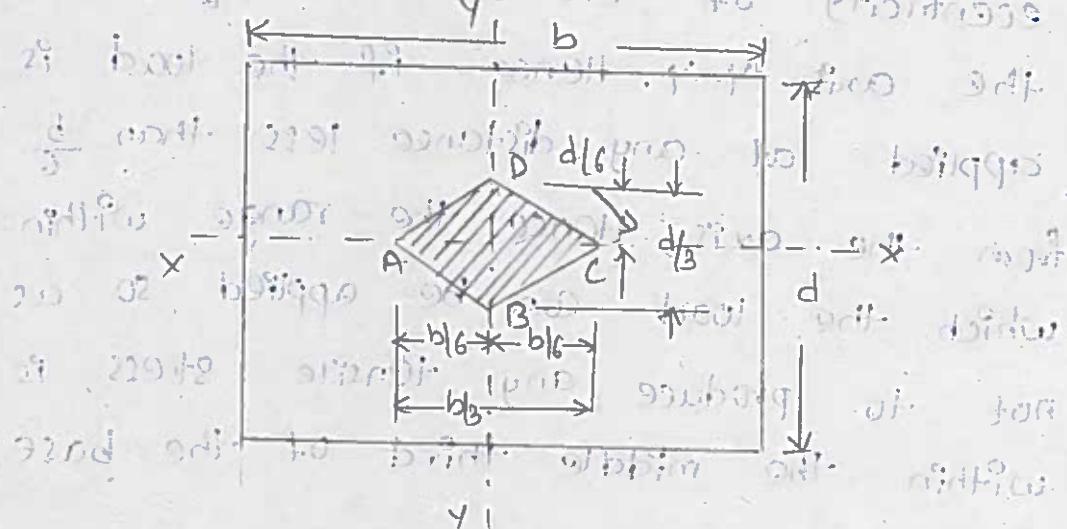
$$= 8 (1 - 0.3) = 5.6 \text{ N/mm}^2$$

Resultant stress for unsymmetrical columns with eccentric loading:

In case of unsymmetrical columns which are subjected to eccentric loading, first the centre of gravity of the unsymmetrical section is determined. Then the moment of inertia of the section about the axis passing through the C.G. is calculated. After that the distances between the corners of the section and its C.G. is obtained. By using the values of the moment of inertia and distances of the corner from the C.G. of the section, the stresses on the corners are then determined.

Middle third rule for Rectangular sections [i.e., kernel of section]

The cement concrete columns are weak in tension. Hence the load must be applied on these columns in such a way that there is no tensile stress anywhere in the section.



Consider a rectangular section of width 'b' and depth 'd' as shown in fig. Let this section is subjected to a load which is

eccentric to the axis Y-Y.

P = Eccentric load acting on the column.

e = Eccentricity of the load

A = Area of the section.

from E.S.U., we have the min stress as

$$\sigma_{\min} = \frac{P}{A} \left[1 - \frac{6xe}{b} \right] \quad \text{--- (1)}$$

If σ_{\min} is -ve, then stress will be tensile. But if σ_{\min} is zero (or +ve) then there will be no tensile stress along the width of the column.

Hence for no tensile stress along the width of the column,

$$\sigma_{\min} \geq 0$$

$$\frac{P}{A} \left[1 - \frac{6xe}{b} \right] \geq 0 \quad \text{or} \quad \left[1 - \frac{6xe}{b} \right] \geq 0$$

$$1 \geq \frac{6xe}{b} \quad \text{or} \quad \frac{b}{6} \geq e$$

$$e \leq \frac{b}{6}$$

The above result shows that the eccentricity 'e' must be less than or equal to $\frac{b}{6}$: hence the greatest eccentricity of the load is $\frac{b}{6}$ from the axis Y-Y. Hence if the load is applied at any distance less than $\frac{b}{6}$ from the axis, hence the range within which the load can be applied so as not to produce any tensile stress, is within the middle third of the base.

rhombus ABCD whose diagonals are $\frac{b}{3}$ and $\frac{d}{3}$. This figure ABCD within which the load may be applied anywhere so as not to produce tensile stress in any part of the entire rectangular section, is called the core or kernel of the section.

Middle Quarter Rule for Circular Sections
(i.e. kernel of section)

Consider a circular section of dia 'd' as shown in fig.

This section is subjected to a load which is eccentric to the axis Y-Y.

P = eccentric load

e = eccentricity of the load

direct stress,

$$\sigma_0 = \frac{P}{A} = \frac{P}{\pi d^2} = \frac{4P}{\pi d^2}$$

$$M = P \times e$$

Bending stress (σ_b) is given by,

$$\frac{M}{I} = \frac{\sigma_b}{y} \quad (\text{or}) \quad \sigma_b = \frac{M \times y}{I}$$

Max. bending stress will be when

$$y = \pm \frac{d}{2}$$

Max. bending stress is given by,

$$\sigma_b = \frac{M}{I} \times \left[\pm \frac{d}{2} \right] = \pm \frac{P \times e \times \frac{d}{2}}{\pi/6 \times d^4} = \pm \frac{32 \times P \times e}{\pi d^3}$$

Now min stress is given by,

$$\sigma_{\min} = \sigma_0 - \sigma_b$$

$$= \frac{4P}{\pi d^2} - \frac{32 P \times e}{\pi d^3}$$

for no tensile stress, $\sigma_{min} \geq 0$

$$\frac{4P}{\pi d^2} - \frac{32Pe^2}{\pi d^3} \geq 0 \text{ or } \frac{4P}{\pi d^2} \left(1 - \frac{8e}{d}\right) \geq 0$$
$$1 - \frac{8e}{d} \geq 0 \text{ or } 1 \geq \frac{8e}{d} \text{ or } e \leq \frac{d}{8}$$

The above result shows that the eccentricity 'e' must be less than or equal to $\frac{d}{8}$. It means that the load can be eccentric, on any side of the centre of the circle, by an amount equal to $\frac{d}{8}$.

DAMS AND RETAINING WALLS

Types of Dams

there are many types of dams, but the following types of dams are more important

- 1) Rectangular dam
- 2) Trapezoidal dam having
 - a) water face vertical, and
 - b) water face inclined.

A trapezoidal dam as compared to rectangular dam is economical and easier to construct. Hence these days trapezoidal dams are mostly constructed.

Rectangular Dam

Shows a rectangular dam having water on one of its sides.

h = height of water.

F = force exerted by water on the side of the dam.

w = weight of dam per metre length of side of dam

H = height of dam

b = width of dam

w_0 = weight/density of dam

The forces acting on the dam are:

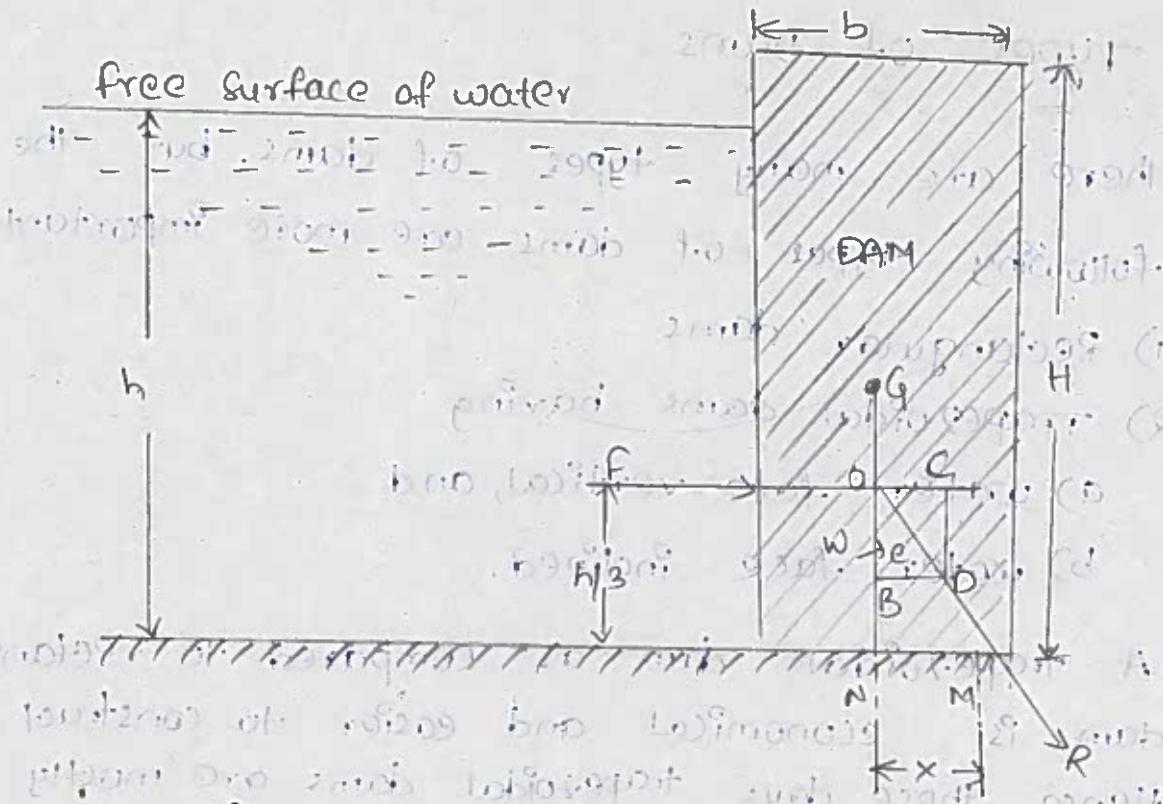
i) the force f due to water in contact with the side of the dam.

The force f is given by

$$F = wAb$$

$$\therefore F = w \times (h \times 1) \times \frac{h}{2} \quad \left[\because A = h \times 1 \text{ and } b = \frac{h}{2} \right]$$

$$2.11 \times 10^3 = \frac{\omega_0 \times b^2}{2}$$



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The force F will be acting horizontally at a height of $\frac{h}{3}$ above the base as shown in fig.

(ii) the weight w of the dam: the weight of the dam is given by.

$$\begin{aligned} w &= \text{weight density of dam} \times \text{volume of dam} \\ &= \omega_0 \times [\text{Area of dam}] \times h \\ &= \omega_0 \times b \times h \end{aligned}$$

There are only two forces acting on the dam. The force F is produced to intersect the line of action of the w at O . Take $OC = F$ and $OB = w$ to some scale. Then the diagonal OD will represent the resultant R to the same scale.

$$\therefore \text{Resultant } R = \sqrt{F^2 + w^2} \quad \text{--- (1)}$$

$$\tan \theta = \frac{BO}{OB} = \frac{F}{w} \quad \text{--- (2)}$$

The horizontal distance between the line of action of w and the point through which the resultant cuts the base.

The diagonal OP represents the resultant of F and w . Let the diagonal OP is extend so that it cuts the base of the dam at point M . Also extend the line OB so that it cuts the base at point N .

Let $x = \text{Distance } MN$.

The distance x is obtained from similar triangles OBD and ONM is given below.

$$\frac{NM}{ON} = \frac{BD}{OB}$$

$$\frac{x}{(h/3)} = \frac{f}{w} \quad [\because \text{Distance } ON = h/3, BD = OC \\ = f \text{ and } OB = w]$$

$$x = \frac{f}{w} \times \frac{h}{3}$$

The distance x can also be calculated by taking moments of all forces (here the forces F and w) about the point M .

$$fx \frac{h}{3} = wx \cdot b$$

$$x = \frac{f}{w} \times \frac{h}{3}$$

$B.M.$

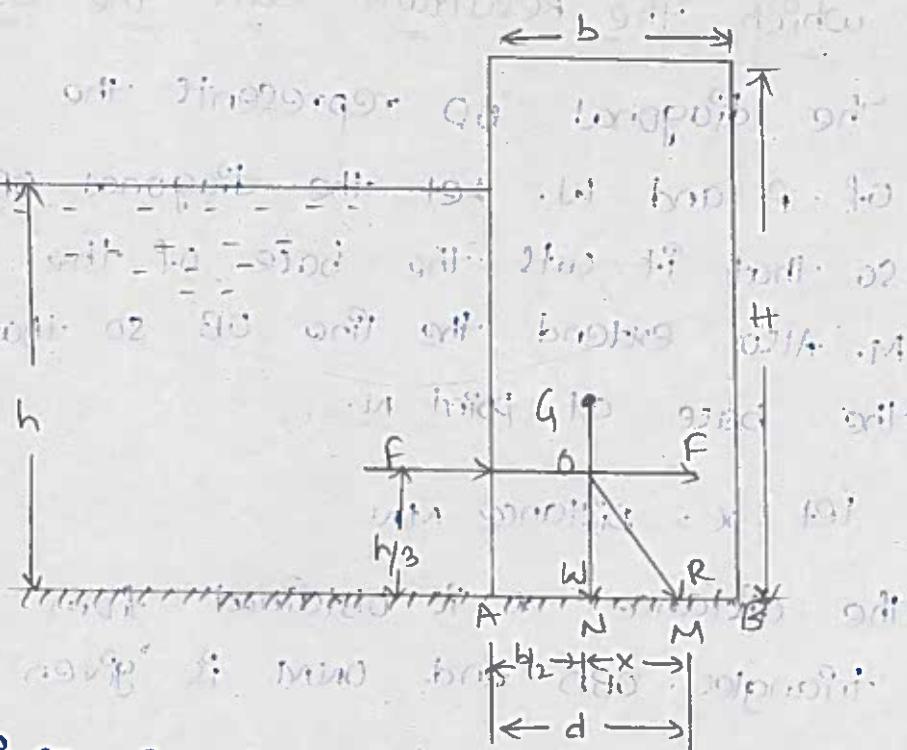
$D.W. \rightarrow M$



$\Sigma M_1 = 0$

$\Sigma M_2 = 0$

Stresses Across the section of a Rectangular Dam



- i) The force F due to water at a height of $\frac{h}{3}$ above the base of the dam.
- ii) The weight w of the dam at the c.g. of the dam.

$$w = \frac{f}{h} \times \frac{h}{3}$$

d = Distance $AM = AN + NM$ x width

$$= \frac{b}{2} + \frac{f}{h} \times \frac{h}{3}$$

\therefore eccentricity, e = Distance x (or Distance NM)

$$= AM - AN = d - \frac{b}{2}$$

Now the moment on the base section.

$$= w \times \text{eccentricity}$$

$$= w \cdot e$$

$$M = w \cdot e$$

$$\frac{M}{I} = \frac{6b}{45} \quad \text{--- (1)}$$

$$= \frac{1 \times b^3}{12}$$

$$= \frac{b^3}{12}$$

Sub the values in eqn (1), we get unit-3, pg.no-12/16

$$\frac{W \cdot e}{(b^3/12)} = \frac{\sigma_b}{(\pm b/2)}$$

$$\sigma_b = \pm W \cdot e \frac{b}{2} \times \frac{12}{b^3} = \pm \frac{6 W \cdot e}{b^2}$$

The bending stress at point B

$$= \frac{6 W \cdot e}{b^2}$$

bending stress at point A

$$= - \frac{6 W \cdot e}{b^2}$$

But the direct stress on the base section due to direct load is given by

$$\sigma_0 = \frac{\text{weight of dam}}{\text{Area of base}} = \frac{W}{b \times l} = \frac{W}{b}$$

Total stress across the base at A,

$$\sigma_{\min} = \sigma_0 + \text{Bending stress at point A}$$

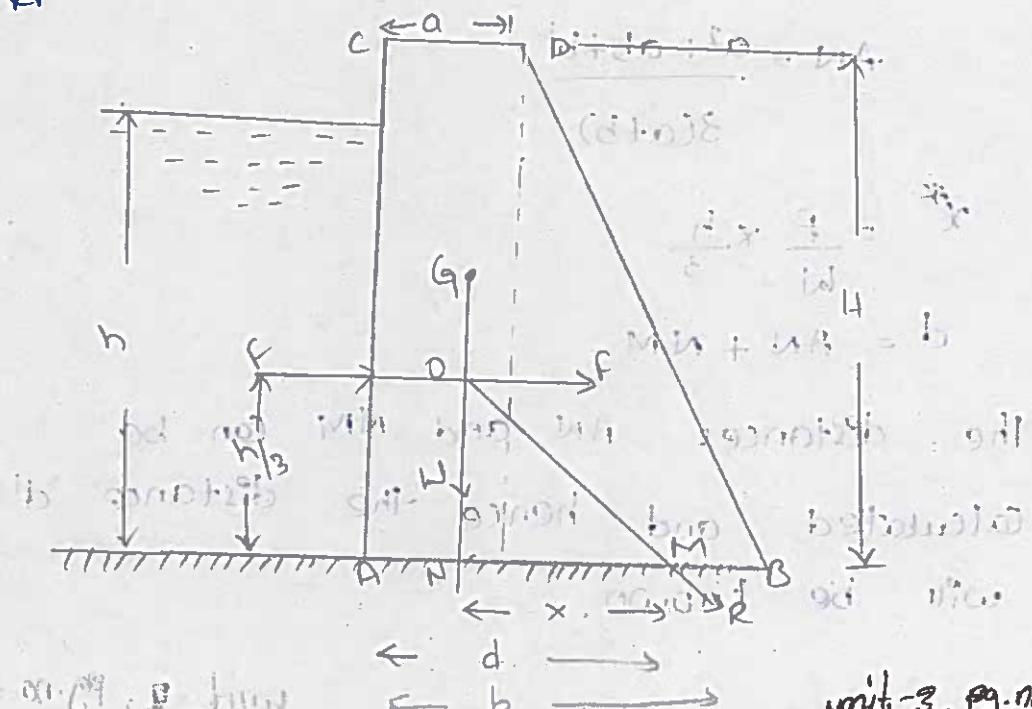
$$= \frac{W}{b} - 6 \frac{W \cdot e}{b^2}$$

$$= \frac{W}{b} \left[1 - \frac{6 \cdot e}{b} \right]$$

If the value of σ_{\min} is (-ve), this means that at the point A the stress is tensile.

Trapezoidal Dam having water face vertical, \therefore Water pressure is uniform

Pr



Now the forces acting on the dam
are

i) $F = \text{Force exerted by water}$

$$= w \times A \times h = w \times (h \times 1) \times \frac{h}{2} = w \times \frac{h^2}{2}$$

The force F will be acting horizontally
at a height of $h/3$ above the base.

(ii) w - weight of dam per metre length
of dam

$$= \text{weight density of dam} \times (\text{Area of C.G.}) \times r$$

$$= w_0 \times \left[\frac{a+b}{2} \right] \times h \times 1$$

$$= w_0 \times \frac{(a+b)}{2} \times h$$

i.e., Area of rectangle \times Distance of C.G.
of rectangle from AC + Area of triangle
 \times Distance of C.G. of triangle from AC =
Total area of trapezoidal \times Distance AN

$$(a \times h) \times \frac{a}{2} + \frac{(b-a) \times h}{2} \left[\frac{a+b}{3} \right] = \left[\frac{a+b}{2} \right] \times h \times AN$$

from the above, equ. distance AN can
be calculated.

(ii) the distance AN can also be calculated
by using the relation given below.

$$AN = \frac{a^2 + ab + b^2}{3(a+b)}$$

$$x^* = \frac{F}{w} \times \frac{h}{3}$$

$$d = AN + NM$$

The distances AN and NM can be
calculated and hence the distance 'd'
will be known.

Now the eccentricity, e

$e = d - \text{half the base width of the dam}$

$$= d - \frac{b}{2}$$

Then the total stress across the base of the dam at point B,

$$\sigma_{\max} = \frac{W}{b} \left[1 + \frac{6 \cdot e}{b} \right]$$

and the total stress, across the base at A,

$$\sigma_{\min} = \frac{W}{b} \left[1 - \frac{6 \cdot e}{b} \right]$$